Anomalies and divergences in (super)gravity

Julio Parra-Martinez

based on:
Z. Bern, A. Edison, D. Kosower, JPM [1706.01486],
Z. Bern, JPM, R. Roiban [1712.03928],
Z. Bern, D.Kosower, JPM [In preparation]

UCLA The Mani L. Bhaumik Institute for Theoretical Physics

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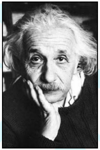
Gravity as an Efective Field Theory



- Gravity always shamed for being non-renormalizable.
- Perfectly fine EFT, valid for very large range of energies.
- EFT lore (totallitarian principle):

Everything that is allowed (by symmetry) is compulsory.

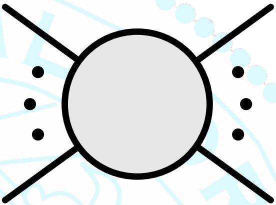
$$\mathcal{L} = M_{\rm pl}^2 R + c_2 R^2 + c_3 R^3 + \cdots c_{k,n} D^k R^n + \cdots \,, \qquad c_{n,k} \sim \frac{1}{\Lambda^{k+2n-4}}$$

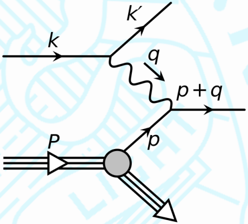
- Apearance of higher-dim. operators, running controlled by divergences.
- Questions unanswered:
  - When do divergences exactly arise and under which circumstances?
  - Do they always arise?

How do we answer these questions?

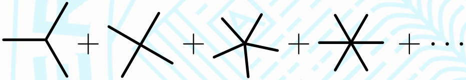
Direct computation of Scattering Amplitudes

- Give probability of scattering event
- On-shell quantities (i.e., do not depend on choice of variables)
- Calculation techniques developed over years
- Provide a window to the ultraviolet (UV)





But calculating gravity Feynman diagrams is hard. . .

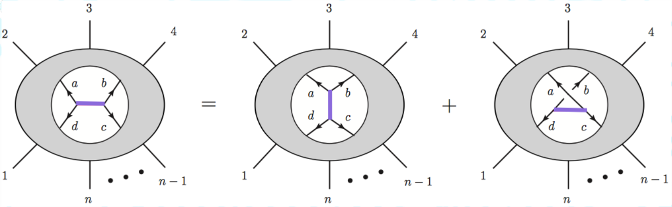
$\mathcal{L}_g \supset$ 

Double-Copy construction (a little miracle)

Yang-Mills and gravity integrands simply related

$$\mathcal{A} = \int \frac{d^D\ell}{(2\pi)^D} \sum_{i\in\Gamma} \frac{1}{S_i} \frac{n_i\, c_i}{\prod_{\alpha\in i} D_\alpha} \quad \to \quad \mathcal{M} = \int \frac{d^D\ell}{(2\pi)^D} \sum_{i\in\Gamma} \frac{1}{S_i} \frac{n_i\, \tilde{n}_i}{\prod_{\alpha\in i} D_\alpha}$$

if we can arrange $\quad c_1 + c_2 + c_3 = 0 \quad \leftrightarrow \quad n_1 + n_2 + n_3 = 0$ [BCJ]



Yang-Mills integrands are much easier to construct: unitarity, recursion...

Supergravity in the UV: a summary

- These techniques allowed recent progress in supergravity (SUGRA)
- UV cancellations beyond what was previouly expected by symmetry

$\mathcal{N}$ \ L	1	2	3	4	5	$\cdots$	7
0	0	∞	$\cdots$				
4	0	0	0	∞	$\cdots$		
5	0	0	0	0	?	$\cdots$	
8	0	0	0	0	soon	$\cdots$	?

[Bern, Carrasco, Davies, Dennen, Dixon, Johansson, Kosower, Nohle, Roiban, Smirnov^2. . .]

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[Bern, Carrasco, Davies, Dennen, Dixon, Johansson, Kosower, Nohle, Roiban, Smirnov[2]. . .]

+ half-maximal supergravity at $L = 2$ in five dimensions.

[Tourkine,Vanhove]

Big question: Why are they finite? Symmetry?

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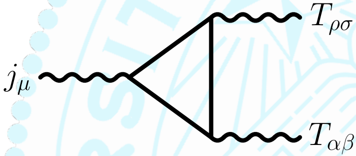
Big question: Why are they finite? Symmetry?

I will focus on the divergent cases and their relation to anomalies.

Anomalies

- Reminder: Symmetries play an important role in field theory
- *Classical* symmetries can be broken by quantum corrections
- Ward identity $\partial_\mu J^\mu = 0$ $\rightarrow$ selection rule for the S-matrix.
- Broken at loop level



$$\sim \quad \Gamma_{1-\text{loop}} \supset \int \mathrm{d}^4x \, RR^* \frac{\partial_\mu A^\mu}{\Box}$$

- Not a problem if they can be removed by a local counterterm.

Evanescent effects & the trace anomaly

One-loop finiteness in gravity

One loop graviton amplitudes finite because ['t Hooft, Veltman]

$$E_4 = R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

is evanescent (has vanishing matrix elements *in four dimensions*).

Divergence is not numerically zero

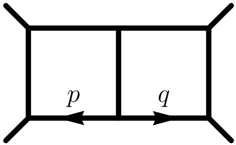
$$\mathcal{M}^{1-\text{Loop}}\Big|_{\text{div}} = -\frac{1}{(4\pi)^2}\frac{1}{360\epsilon}\frac{a-c}{2}\mathcal{M}_{R^2}$$

related to the trace anomaly [Duff; Christensen, Duff; Hawking, Perry; ...]

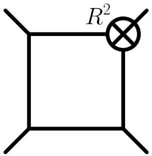
$$T^{\mu}{}_{\mu} = -\frac{1}{(4\pi)^2}\frac{1}{360}(a\,E_4 - c\,W^2) + \cdots$$

Effects at higher loops

Evanescent counterterms contaminate divergence



$\sim (\mu_R^2)^{2\epsilon} \times \frac{c_1}{\epsilon}$



$\sim (\mu_R^2)^{\epsilon} \times \frac{c_2}{\epsilon}$

$$\to \mathcal{M}\Big|_{\text{div}} \sim (c_1 + c_2)\frac{1}{\epsilon} + (2c_1 + c_2)\log\mu_R^2 + \cdots$$

coefficient of $\frac{1}{\epsilon}$ and log disconnected. [Bern, Cheung, Chi, Davies, Dixon, Nohle]

$$\mathcal{M}_{4,\text{pure G.}}^{2-loop} = \left(\frac{1}{\epsilon}\frac{209}{24} - \frac{1}{4}\log\mu_R^2\right)\mathcal{M}_{R^3} + \cdots$$ [Goroff, Sagnotti; Van de Ven]

$$\mathcal{M}_{4,\mathcal{N}=1}^{2-loop} = \left(\frac{1}{\epsilon}\frac{341}{32} - 0\,\log\mu_R^2\right)\mathcal{M}_{R^3} + \cdots$$ [Bern, Chi, Dixon, Edison]

simple formula for scale dependence $-\frac{N_B - N_F}{8}\log\mu_R^2\,\mathcal{M}_{R^3}$

Duality symmetries and anomalies in $\mathcal{N} = 4$ SUGRA

$\mathcal{N} = 4$ SUGRA and duality symmetry

- Double copy: $(\mathcal{N} = 4 \text{ SUGRA}) \equiv (\mathcal{N} = 4 \text{ SYM}) \otimes (\text{pure YM})$
- States: $\Phi^+ = \Phi \otimes g^+ \quad \Phi^- = \Phi \otimes g^-$, with $\Phi = (\mathcal{N} = 4 \text{ multiplet})$
- Classification of amplitudes: $M_{n,k}^{(n_+,n_-)}$
- Conserved charge: $\sum q_i = h_i(\text{YM}) - h_i(\text{SYM}) = 0 \to n_- = k + 2$

$$\mathcal{M}_{4,0}^{(0,4)} = 0 \quad \supset \quad \mathcal{M}(h_1^{--}\, h_2^{--}\, t_3\, t_4)$$
$$\mathcal{M}_{4,0}^{(1,3)} = 0 \quad \supset \quad \mathcal{M}(h_1^{++}\, h_2^{--}\, h_3^{--}\, t_4)$$

- $U(1)$ can be identified with a subgroup of the $\mathrm{SU}(1,1)$ duality symmetry.

[Carrasco, Kallosh, Roiban, Tseytlin]

Anomaly and evanescent contributions at one loop

Same amplitudes non-vanishing at one-loop due to anomaly

[Carrasco, Kallosh, Roiban, Tseytlin]

$$\bar{\mathcal{M}}^{(4,0)}_{\text{1-loop}} \neq 0, \quad \bar{\mathcal{M}}^{(3,1)}_{\text{1-loop}} \neq 0, \quad \bar{\mathcal{M}}^{(5,0)}_{\text{1-loop}} \neq 0, \quad \bar{\mathcal{M}}^{(0,5)}_{\text{1-loop}} \neq 0,$$

Non-anomalous amplitude contains [Bern, Edison, Kosower, JPM]

$$M^{(2,2)}_{4,0} = M_{R^2} + \cdots$$

Evanescent contribution and anomalous pieces have same structure

$$A_{SYM} \otimes A_{F^3}$$

Q: Could it be that both the anomaly and the evanescent pieces can be removed by local counterterms?

Cancellation of anomalous amplitudes

- We calculated all one-loop anomalous amplitudes for $n = 3, 4, 5$.
- We found an “inverse-soft” formula for n-point amplitudes with $n_- > 2$

  [Bern, JPM, Roiban]

- We calculated 2-loop 4-point anomalous amplitudes

  [Bern, Kosower, JPM in preparation]

- All of them and evanescent cancelled by adding finite local counterterm

$$S_{\text{ct.}} \propto -\bar{\tau}(R^+)^2 + \tau(R^-)^2 + \text{ SUSY} = e^{-\phi}E_4 - b\,R \wedge R + \text{ SUSY}$$

- Double-copy for higher dimensional operators [Broedel, Dixon; He, Zhang]

$$A_{YM} \otimes A_{F^3} \sim M_{\phi^n R^2}$$

Why such operator?

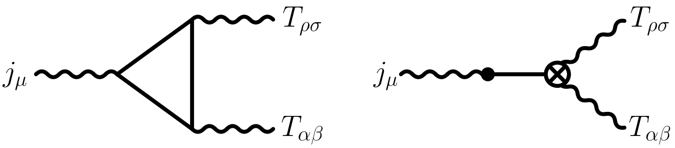
Anomaly cancellation in string theory requires [Green-Schwarz]

$$H = dB + c_1\omega_{3A} + c_2\omega_{3L} \quad \text{and} \quad B \wedge F^{\frac{d-2}{2}}$$

which in 4D produces [Dine, Seiberg, Witten; Atick, Dixon, Sen]

$$H^2 \supset \omega_{3L} \wedge *dB = \omega_{3L} \wedge db = -b\,R \wedge R + d(\cdots)$$

so cancellation mechanism appears to be $D = 4$ Green-Schwarz.



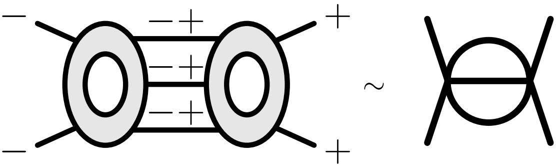
Operator necessary for $\mathcal{N} = 4$ SUGRA as low-energy limit of a string theory! [Gregori, Kiritsis, Kounnas, Obers, Petropoulos, Pioline]

Four-loop divergence

- Divergence found at four loops [Bern, Davies, Dennen, Smirnov^2]

$$\mathcal{M}_4^{4-\text{loop}}\Big|_{\text{div}} = \frac{1}{\epsilon}\frac{(1-264\zeta_3)}{144} st\mathcal{A}_4^{\text{tree}}(\mathcal{O}^{(2,2)} + \mathcal{O}^{(4,0)} + \mathcal{O}^{(3,1)}).$$

- Anomalous amplitudes contribute in cuts of non-anomalous ones



- All cuts of anomalous amplitudes vanish or are cancelled in 4D numerators $\mathcal{O}(\epsilon) \rightarrow$ should be suppressed w.r.t non-anomalous!

[Bern, Kosower, JPM in preparation]

- Divergence should be reanalyzed.

Summary

- A better understanding of the divergence structure of (super)gravity is needed
- In divergent $\mathcal{N} = 4$ SUGRA anomaly and evanescent contributions are closely intertwined
- Effects of both in large classes of amplitudes can be removed by adding a local counterterm, perhaps the divergence?
- Many cancellations calling for an explanation!

Thank you!