Cancellation of anomalous amplitudes in $\mathcal{N} = 4$ supergravity

QCD Meets Gravity

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December 11, 2017

Supergravity in the UV

- Recent progress in supergravity (SUGRA) calculations
- Allowed by the double-copy construction
- UV cancellations beyond what was previouly expected

$\mathcal{N}$ \ L	1	2	3	4	5	$\cdots$	7
0	0	∞	$\cdots$				
4	0	0	0	∞	$\cdots$		
5	0	0	0	0	?	$\cdots$	
8	0	0	0	0	soon	$\cdots$	?

[Bern, Carrasco, Davies, Dennen, Dixon, Johansson, Kosower, Nohle, Roiban, Smirnov^2. . .]

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+ half-maximal supergravity at $L = 2$ in five dimensions.

[Tourkine, Vanhove]

Big questions: Are they finite?
If so, why?

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I will focus on the divergent cases and their relation to anomalies.

Evanescent effects & the conformal anomaly

One-loop finiteness in gravity

One loop graviton amplitudes finite because ['t Hooft, Veltman]

$$E_4 = R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

is evanescent (has vanishing matrix elements *in four dimensions*).

Divergence is not numerically zero

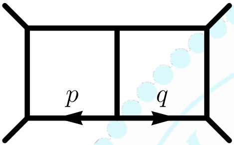
$$\mathcal{M}^{1-\text{Loop}}\Big|_{\text{div}} = -\frac{1}{(4\pi)^2}\frac{1}{360\epsilon}\frac{a-c}{2}\mathcal{M}_{R^2}$$

related to the conformal anomaly [Duff; Christensen, Duff; Hawking, Perry; ...]

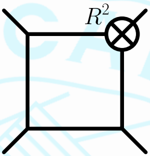
$$T^{\mu}{}_{\mu} = -\frac{1}{(4\pi)^2}\frac{1}{360}\left(a\,E_4 - c\,W^2\right) + \cdots$$

Effects at higher loops

Evanescent counterterms contaminate divergence



$\sim (\mu_R^2)^{2\epsilon} \times \frac{c_1}{\epsilon}$



$\sim (\mu_R^2)^{\epsilon} \times \frac{c_2}{\epsilon}$

$$\to \mathcal{M}\big|_{\text{div}} \sim (c_1 + c_2)\frac{1}{\epsilon} + (2c_1 + c_2)\log \mu_R^2 + \cdots$$

coefficient of $\frac{1}{\epsilon}$ and log disconnected. [Bern, Cheung, Chi, Davies, Dixon, Nohle]

$$\mathcal{M}_{4,\text{pure G.}}^{2-loop} = \left(\frac{1}{\epsilon}\frac{209}{24} - \frac{1}{4}\log \mu_R^2\right)\mathcal{M}_{R^3} + \cdots$$ [Goroff, Sagnotti; Van de Ven]

$$\mathcal{M}_{4,\mathcal{N}=1}^{2-loop} = \left(\frac{1}{\epsilon}\frac{341}{32} - 0\,\log \mu_R^2\right)\mathcal{M}_{R^3} + \cdots$$ [Bern, Chi, Dixon, Edison]

simple formula for scale dependence $-\frac{N_B - N_F}{8}\log \mu_R^2\,\mathcal{M}_{R^3}$

Duality symmetries and anomalies in SUGRA

Duality "symmetry" in Supergravity

- Scalars in $\mathcal{N} \geq 4$ SUGRA parameterize a coset G/H
  - ϕ^{ABCD} in $\mathcal{N} = 8 \rightarrow \mathrm{E}_{7(7)}/\mathrm{SU}(8)$
  - τ in $\mathcal{N} = 4 \rightarrow \mathrm{SU}(1,1)/\mathrm{U}(1)$
- $H \equiv R$-symmetry, acts linearly on everything, e.m. duality on the vectors. $\rightarrow$ on-shell symmetry, no gauge invariant current!
- Two points of view for the scalars:
  - G nonlinearly realized
  - G acts linearly and H is gauged
- "Anomalies" in H studied long ago [Marcus]:
  No anomalies for $\mathcal{N} \geq 5$, anomaly in $\mathcal{N} = 4$
- Important for understanding UV behaviour (counterterms)
  [Green, Russo, Vanhove; Broedel, Dixon; Bossard, Howe, Stelle; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger; ...]

Two supermultiplets

$$\Phi^{+} = h^{++} + \eta^{A}\psi_{A}^{+} + \frac{1}{2!}\eta^{A}\eta^{B}A_{AB}^{+} + \frac{1}{3!}\eta^{A}\eta^{B}\eta^{C}\epsilon_{ABCD}\,\chi^{+D} + \frac{1}{4!}\eta^{A}\eta^{B}\eta^{C}\eta^{D}\epsilon_{ABCD}\,\bar{t}$$

$$\Phi^{-} = t + \eta^{A}\chi_{A}^{-} + \frac{1}{2!}\eta^{A}\eta^{B}A_{AB}^{-} + \frac{1}{3!}\eta^{A}\eta^{B}\eta^{C}\epsilon_{ABCD}\,\psi^{-D} + \frac{1}{4!}\eta^{A}\eta^{B}\eta^{C}\eta^{D}\epsilon_{ABCD}\,h^{--}$$

Different scalars related by

$$\tau = i - i\log(1-t) = b + ie^{-\varphi}$$

In the double copy $\quad \Phi^{+} = \Phi \otimes g^{+} \quad \Phi^{=} = \Phi \otimes g^{-}$

$$\Phi = g^{+} + \eta^{A}\psi_{A} + \frac{1}{2!}\eta^{A}\eta^{B}\phi_{AB} + \frac{1}{3!}\eta^{A}\eta^{B}\eta^{C}\epsilon_{ABCD}\,\bar{\psi}_{D} + \frac{1}{4!}\eta^{A}\eta^{B}\eta^{C}\eta^{D}\epsilon_{ABCD}\,g^{-}$$

Classification of amplitudes $\mathrm{N}^{k}\mathrm{MHV}^{(n_{+},n_{-})}$:

$$M_{n,k}^{(n_{+},n_{-})} \equiv M_{n,k}(\Phi_{1}^{+},\ldots,\Phi_{n_{+}}^{+},\Phi_{n_{+}+1}^{-},\ldots,\Phi_{n}^{-})$$

Tree-level U(1) symmetry

Conserved charge: $q = h(\mathrm{YM}) - h(\mathrm{SYM})$

$$q(h^{\pm\pm}) = 0 \quad q(\psi^{\pm}) = \pm\frac{1}{2} \quad q(A^{\pm}) = \pm 1 \quad q(\chi^{\pm}) = \pm\frac{3}{2} \quad q(t,\bar{t}) = (-2,2)$$

Tree-level selection rule $\sum_i q_i = 0$

$$n_+ = n - k - 2, \quad n_- = k + 2$$

For instance

$$\mathcal{M}_{4,0}^{(0,4)} = 0 \quad \supset \quad \mathcal{M}(h_1^{--}\, h_2^{--}\, t_3\, t_4)$$
$$\mathcal{M}_{4,0}^{(1,3)} = 0 \quad \supset \quad \mathcal{M}(h_1^{++}\, h_2^{--}\, h_3^{--}\, t_4)$$

Can be identified with a subgroup of the SU(1,1) duality symmetry.

[Carrasco, Kallosh, Roiban, Tseytlin]

Anomaly at one loop

Same amplitudes non-vanishing at one-loop due to anomaly

[Carrasco, Kallosh, Roiban, Tseytlin]

$$\bar{\mathcal{M}}^{(4,0)}_{\text{1-loop}} \neq 0, \quad \bar{\mathcal{M}}^{(3,1)}_{\text{1-loop}} \neq 0, \quad \bar{\mathcal{M}}^{(5,0)}_{\text{1-loop}} \neq 0, \quad \bar{\mathcal{M}}^{(0,5)}_{\text{1-loop}} \neq 0,$$

from soft limits

$$\mathcal{M}^{(0,n)}_{\text{1-loop}} = i\left(\frac{\kappa}{2}\right)^n (n-3)!\,\delta^{(8)}(Q) \quad \supset \quad \mathcal{M}(h^{--}\,h^{--}\,t^{n-2})$$

corresponding to a term in the effective action

$$\Gamma^{\text{local}}_{\text{anom}} \propto \bar{\tau}(R^+)^2 - \tau(R^-)^2 + \text{ SUSY} = e^{-\phi}E_4 - b\,R\wedge R + \text{ SUSY}$$

but the rest of the anomalous amplitudes are nonlocal.

Evanescent contributions at one loop

- Non-anomalous amplitude contains [Bern, Edison, Kosower, JPM]

$$M_{4,0}^{(2,2)} = M_{R^2} + \cdots$$

- Evanescent contribution and anomalous pieces have same structure

$$A_{SYM} \otimes_{\mathrm{KLT}} A_{F^3}$$

and originate in same rational terms of pure YM.

Q: Could it be that both the anomaly and the evanescent pieces can be removed by local counterterms?

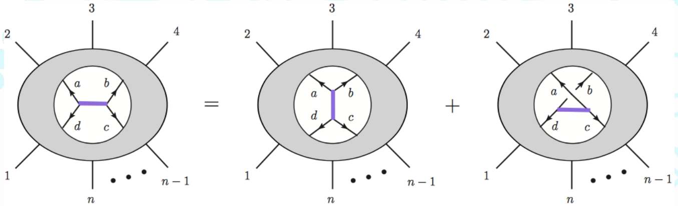
Our calculation

Double-Copy

We use $(\mathcal{N}=4\ \text{SUGRA}) \equiv (\mathcal{N}=4\ \text{SYM}) \otimes (\text{pure YM})$

$$\mathcal{A} = \int \frac{d^D\ell}{(2\pi)^D} \sum_{i\in\Gamma} \frac{1}{S_i} \frac{n_i\, c_i}{\prod_{\alpha\in i} D_\alpha} \quad \to \quad \mathcal{M} = \int \frac{d^D\ell}{(2\pi)^D} \sum_{i\in\Gamma} \frac{1}{S_i} \frac{n_i\, \tilde{n}_i}{\prod_{\alpha\in i} D_\alpha}$$

if we can arrange $\quad c_1 + c_2 + c_3 = 0 \quad \leftrightarrow \quad n_1 + n_2 + n_3 = 0$ [BCJ]



$$M_n = \sum_{\sigma\in S_n/Z_n} n_\sigma^{\mathcal{N}=4} A^{\text{YM}}_{n,1-\text{loop}}(\sigma) \quad \text{for} \quad n = 4, 5$$

[Bern, Boucher-Veronneau, Johansson]

Amplitudes

We (re)calculate all one-loop anomalous amplitudes for $n = 3, 4, 5$.

$$M_{n,0}^{(1,n-1)} = -i \sum_{r=2}^{n-2} \frac{[1\,r]\,\langle r\,n-1\rangle\,\langle r\,n\rangle}{\langle 1\,r\rangle\,\langle 1\,n-1\rangle\,\langle 1\,n\rangle}\,\delta^{(8)}(Q)\,,$$

$$M_{5,0}^{(2,3)} = -i\varepsilon(1,2,3,4)\frac{\langle 3\,4\rangle^2\,\langle 4\,5\rangle^2\,\langle 5\,3\rangle^2}{\prod_{i<j}\langle i\,j\rangle}\,\delta^{(8)}(Q)\,,$$

$$M_{5,0}^{(4,1)} = -i\frac{s_{12}s_{34}\delta^{(8)}(Q)}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle}\left(\frac{2\,[3\,2]\,[2\,4]^2\,[4\,5]}{[1\,2]\,[1\,4]\,\langle 3\,5\rangle}\right.$$
$$\left.+\frac{[3\,4]^2\,\langle 1\,3\rangle^2\,[1\,4]^2+[2\,5]^2\,\langle 1\,2\rangle^2\,[1\,5]^2}{\langle 1\,2\rangle\,\langle 1\,4\rangle\,\langle 3\,4\rangle\,\langle 3\,5\rangle\,\langle 2\,5\rangle}\right)+(2\leftrightarrow 3)\,,$$

$$M_{5,0}^{(5,0)} = i\sum_{i<j}\frac{(\widehat{\gamma}_{ij})^2}{s_{ij}}\,\delta^{(8)}(Q) \qquad \widehat{\gamma}_{12} = \frac{[1\,2]^2\,[3\,4]\,[4\,5]\,[3\,5]}{\varepsilon(1,2,3,4)}\,.$$

All nonlocal except the class $\quad M_{n,0}^{(0,n)} = i\,(n-3)!\,\delta^{(8)}(Q)$

Inverse-soft construction

[Arkani-Hamed, Cachazo, Cheung, Kaplan; Boucher-Veronneau, Larkoski; Nandan, Wen]

- n-point with $n_- > 2$ given by inverse-soft of the local amplitudes

$$M_{n,0}^{(n_+,n_-)} = i(n_- - 3)!\, S[M]\, \delta^{(8)}(Q) \qquad M = \Phi^- \text{ legs}$$

where $S[M]$ are soft-lifting functions [Dunbar, Ettle, Perkins; Feng, He]

$$S[M] = |\Phi|^{m_1 \dots m_r}_{m_1 \dots m_r} \qquad M = m_1 \dots m_r$$

$$\phi_i^j = \frac{[ij]}{\langle ij\rangle} \text{ for } i \neq j\,, \qquad \phi_i^i = -\sum_{j\neq i} \frac{[ij]\,\langle jx\rangle\,\langle jy\rangle}{\langle ij\rangle\,\langle ix\rangle\,\langle iy\rangle} \qquad \text{[Hodges]}$$

Satisfies correct soft and collinear limits.

- Trivially cancelled by adding finite local counterterm

$$S_{\text{ct.}} = -\Gamma^{\text{local}}_{\text{anom}}\Big|_{\tau=\tau(t)}\,.$$

Remaining cases

- Double-copy for higher dimensional operators [Broedel, Dixon; He, Zhang]

$$A_{YM} \otimes_{\text{KLT}} A_{F^3} \sim M_{\phi^n R^2}$$

- Gives right operator, up to normalization

$$M^{(0,n)}_{n,0,\text{KLT}} = i(n-2)!\delta^{(8)}(Q) \quad \text{vs.} \quad M^{(0,n)}_{n}, 0 = i(n-3)!\,\delta^{(8)}(Q)$$

$$t^n(R^-)^2 \quad \text{vs.} \quad \frac{t^n}{n}(R^-)^2$$

- In all cases we find

$$M_{S_{\text{ct.}}} = -M_{\text{anomalous}}.$$

Evanescent contribution also cancels!

$$S_{\text{ct.}} \propto -e^{-\phi}E_4 + \cdots = -E_4 + \cdots$$

Conclusion:

Both anomalous amplitudes and evanescent contributions can be set to zero by adding a finite local counterterm.

Why such operator?

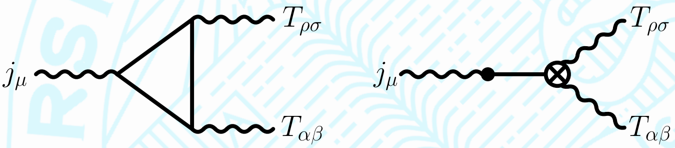
Anomaly cancellation in string theory requires [Green-Schwarz]

$$H = dB + c_1\omega_{3A} + c_2\omega_{3L} \quad \text{and} \quad B \wedge F^{\frac{d-2}{2}}$$

which in 4D produces [Dine, Seiberg, Witten; Atick, Dixon, Sen]

$$H^2 \supset \omega_{3L} \wedge *dB = \omega_{3L} \wedge db = -b\, R \wedge R + d(\cdots)$$

so cancellation mechanism appears to be be $D = 4$ Green-Schwarz.



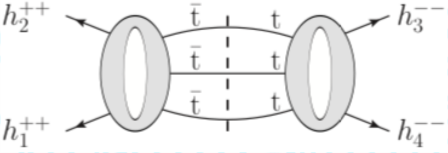
Operator necessary for $\mathcal{N} = 4$ SUGRA as low-energy limit of a string theory! [Gregori, Kiritsis, Kounnas, Obers, Petropoulos, Pioline]

Four-loop divergence

- Divergence found at four loops [Bern, Davies, Dennen, Smirnov^2]

$$\mathcal{M}_4^{4-\text{loop}}\Big|_{\text{div}} = \frac{1}{\epsilon}\frac{(1-264\zeta_3)}{144} st\mathcal{A}_4^{\text{tree}}(\mathcal{O}^{(2,2)} + \mathcal{O}^{(4,0)} + \mathcal{O}^{(3,1)}).$$

- Strange structure: all cuts of anomalous amplitudes vanish in 4D numerators $\mathcal{O}(\epsilon) \to$ should be suppressed w.r.t non-anomalous!
- Anomalous amplitudes contribute in cuts of non-anomalous ones



[Carrasco, Kallosh, Roiban, Tseytlin]

- Divergence should be reanalyzed in presence of counterterm.

Summary

- Effects of trace anomaly on the divergence not physical
- Duality anomaly manifested as nonvanishing amplitudes
- In $\mathcal{N} = 4$ SUGRA anomaly and evanescent contributions are closely intertwined
- Effects of both in large classes of amplitudes can be removed by adding a local counterterm

Work in progress & future questions

- Higher loop anomalous amplitudes and counterterm.
- Inverse-soft for higher dimensional operators?
- Are there any subtleties off-shell?
- Relation to the conformal anomaly in conformal SUGRA?
- What happens to the four loop divergence?