Anomalies and divergences in Supergravity amplitudes

MPI Potsdam Seminar

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Outline

1. Divergences in supergravity and the double-copy
2. Two stories about anomalies
    - Evanescent effects and the conformal anomaly
    - Duality anomalies in supergravity
3. $\mathcal{N} = 4$ supergravity
    - Review
    - Anomalous symmetry & evanescent contributions
    - Cancelling the anomaly?
4. Future directions

Supergravity in the UV

- Recent progress in supergravity (SUGRA) calculations
- Allowed by the double-copy (reviewed in a minute)
- UV cancellations beyond what was previouly expected

$\mathcal{N}$ \ L	1	2	3	4	5	$\cdots$	7
0	0	∞	$\cdots$				
4	0	0	0	∞	$\cdots$		
5	0	0	0	0	?	$\cdots$	
8	0	0	0	0	?	$\cdots$	?

[Bern, Carrasco, Davies, Dennen, Dixon, Johansson, Kosower, Nohle, Roiban, Smirnov^2...]

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$+$ half-maximal supergravity at $L = 2$ in five dimensions.

Big questions: Are they finite? If so, why?

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I will focus on the divergent cases and their relation to anomalies.

Main Tool

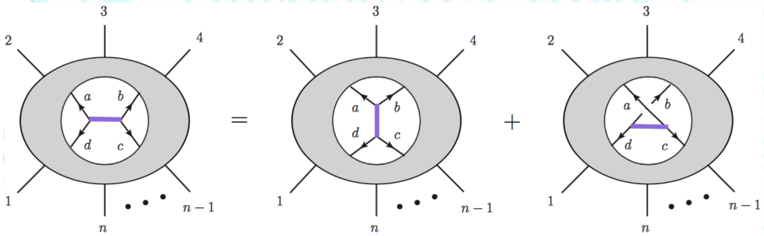
Color-Kinematics duality (BCJ)

Yang-Mills loop integrand organized in terms of trivalent graphs

$$\mathcal{A} = \int \prod_{j=1}^{L} \frac{d^D \ell_j}{(2\pi)^D} \sum_{i \in \Gamma} \frac{1}{S_i} \frac{n_i \, c_i}{\prod_{\alpha \in i} D_\alpha}$$

Color-kinematics duality: pick three graphs

$$c_1 + c_2 + c_3 = 0 \quad \leftrightarrow \quad n_1 + n_2 + n_3 = 0$$



Double-Copy

If color-kinematics dual representation is found, gravity amplitude given by

$$\mathcal{M} = \int \frac{d^D\ell}{(2\pi)^D} \sum_{i\in\Gamma} \frac{1}{S_i} \frac{n_i\,\tilde{n}_i}{\prod_{\alpha\in i} D_\alpha}$$

Kinematic numerators $n_i, \tilde{n}_i$ can belong to different theories, e.g.,

$$\begin{aligned} \mathcal{N} = 8 \text{ SUGRA} \quad &\equiv \quad (\mathcal{N} = 4 \text{ SYM}) \otimes (\mathcal{N} = 4 \text{ SYM}) \\ \mathcal{N} = 4 \text{ SUGRA} \quad &\equiv \quad (\mathcal{N} = 4 \text{ SYM}) \otimes (\mathcal{N} = 0 \text{ SYM}) \end{aligned}$$

Different encarnation at tree level: Kawai-Lewellen-Tye (KLT) relations from String Theory

$$M^{\text{tree}} = \tilde{A}^{\text{tree}}(\alpha) S_{\text{KLT}}(\alpha, \beta) A^{\text{tree}}(\beta)\,,$$

Evanescent effects & the conformal anomaly

One-loop finiteness in gravity

One loop graviton amplitudes are finite because

$$E_4 = R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

is evanescent (has vanishing matrix elements *in four dimensions*).

But the divergence is not numerically zero

$$\mathcal{M}^{1-\text{Loop}}\Big|_{\text{div}} = -\frac{1}{(4\pi)^2}\frac{1}{360\epsilon}\frac{a-c}{2}\mathcal{M}_{R^2}$$

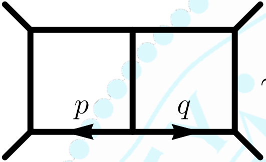
where a and c are the coefficients of the conformal anomaly

$$T^{\mu}{}_{\mu} = -\frac{1}{(4\pi)^2}\frac{1}{360}(a\,E_4 - c\,W^2) + \cdots$$

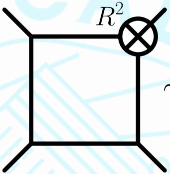
['t Hooft, Veltman]

Effects at higher loops

Evanescent counterterms contaminate divergence



$\sim (\mu_R^2)^{2\epsilon} \times \frac{c_1}{\epsilon}$



$\sim (\mu_R^2)^{\epsilon} \times \frac{c_2}{\epsilon}$

then

$$\mathcal{M}\Big|_{\rm div} \sim (c_1 + c_2)\frac{1}{\epsilon} + (2c_1 + c_2)\log \mu_R^2 + \cdots$$

so coefficient of divergence and renomalization scale dependence are disconnected.

[Bern, Cheung, Chi, Davies, Dixon, Nohle]

Examples

- Pure Einstein Gravity [Goroff, Sagnotti; Van de Ven]

$$\mathcal{M}_4^{2-loop} \simeq \left(\frac{1}{\epsilon}\frac{209}{24} - \frac{1}{4}\log\mu_R^2\right)\mathcal{M}_{R^3} + \text{finite} + \text{IR}$$

- $\mathcal{N} = 1$ SUGRA [Bern, Chi, Dixon, Edison]

$$\mathcal{M}_4^{2-loop} \simeq \left(\frac{1}{\epsilon}\frac{341}{32} - 0\,\log\mu_R^2\right)\mathcal{M}_{R^3} + \text{finite} + \text{IR}$$

- General simple formula for scale dependence

$$-\mathcal{M}_{R^3}\frac{N_B - N_F}{8}\log\mu_R^2$$

in contrast with divergence.

Conclusion: In the presence of evanescent operators, the value of some divergences in dimensional regularization is regulator dependent and can be removed without physical consequences in the scattering amplitudes.

The $\log \mu_R^2$ contains the true scaling behaviour of the theory.

Duality symmetries and anomalies in SUGRA

Duality "symmetry" in Supergravity

- Scalars in $\mathcal{N} \geq 4$ SUGRA parameterize a coset G/H (σ-model), e.g.,
  - $\mathrm{E}_{7(7)}/\mathrm{SU}(8)$ in $\mathcal{N} = 8$
  - $\mathrm{SU}(1,1)/\mathrm{U}(1)$ in $\mathcal{N} = 4$
- *H* is the *R*-symmetry and acts linearly on the whole spectrum
- Two points of view for the scalars:
  - *G* acts nonlinearly
  - *G* acts linearly and *H* is gauged
- Important: *H* acts as electric-magnetic duality on the vectors.
  $\rightarrow$ no gauge invariant current!

Anomalies in duality symmetry

- Posibility of anomalies studied long ago [Marcus]
  - Anomaly polynomials, etc
  - No anomalies for $\mathcal{N} \geq 5$
  - Anomaly in $\mathcal{N} = 4$
- Recent reanalysis from amplitudes perspective [Friedman, Kallosh, Murli, Van Proeyen, Yamada]
  - Available matrix elements but coefficient zero for $\mathcal{N} \geq 5$
- Similar to the conformal anomaly in formulation with scalars. [Nicolai, Townsend]

$\mathcal{N} = 4$ SUGRA

Spectrum and amplitudes

Spectrum consists of two supermultiplets

$$\Phi^+ = h^{++} + \bar\eta^A \psi^+_A + \frac{1}{2!}\bar\eta^A\bar\eta^B A^+_{AB} + \frac{1}{3!}\bar\eta^A\bar\eta^B\bar\eta^C \epsilon_{ABCD}\,\chi^{+D} + \frac{1}{4!}\bar\eta^A\bar\eta^B\bar\eta^C\bar\eta^D \epsilon_{ABCD}\,\bar t$$

$$\Phi^- = t + \bar\eta^A \chi^-_A + \frac{1}{2!}\bar\eta^A\bar\eta^B A^-_{AB} + \frac{1}{3!}\bar\eta^A\bar\eta^B\bar\eta^C \epsilon_{ABCD}\,\psi^{-D} + \frac{1}{4!}\bar\eta^A\bar\eta^B\bar\eta^C\bar\eta^D \epsilon_{ABCD}\,h^{--}$$

scalars $t, \bar t$ linearly related to dilaton ϕ and axion b.

In the double copy

$$\Phi^+ = \Phi \otimes g^+ \quad \Phi^= = \Phi \otimes g^-$$

where $g^\pm$ are the pure YM gluons and Φ is the $\mathcal{N} = 4$ SYM multiplet

$$\Phi = g^+ + \bar\eta^A \psi_A + \frac{1}{2!}\bar\eta^A\bar\eta^B \phi_{AB} + \frac{1}{3!}\bar\eta^A\bar\eta^B\bar\eta^C \epsilon_{ABCD}\,\bar\psi_D + \frac{1}{4!}\bar\eta^A\bar\eta^B\bar\eta^C\bar\eta^D \epsilon_{ABCD}\,g^-$$

Classification of amplitudes $\mathrm{N}^k\mathrm{MHV}^{(n_+,n_-)}$.

Tree-level U(1) symmetry

Conserved charge: $q = h(\text{YM}) - h(\text{SYM})$

$$q(h^{\pm\pm}) = 0 \quad q(\psi^{\pm}) = \pm\frac{1}{2} \quad q(A^{\pm}) = \pm 1 \quad q(\chi\pm) = \pm\frac{3}{2} \quad q(t,\bar{t}) = (-2,2)$$

only amplitudes with

$$n_+ = n - k - 2, \quad n_- = k + 2$$

are nonzero at tree-level, e.g.,

$$\bar{\mathcal{M}}^{(4,0)}_{\text{tree}} = 0 \quad \supset \quad \mathcal{M}(h_1^{++}\, h_2^{++}\, \bar{t}_3\, \bar{t}_4)$$
$$\bar{\mathcal{M}}^{(3,1)}_{\text{tree}} = 0 \quad \supset \quad \mathcal{M}(h_1^{--}\, h_2^{++}\, h_3^{++}\, \bar{t}_4)$$

This symmetry can be identified with a subgroup of the SU(1, 1) duality symmetry. [Carrasco, Kallosh, Roiban, Tseytlin]

Anomaly at one loop

These amplitudes are non-vanishing at one-loop due to anomaly [Carrasco, Kallosh, Roiban, Tseytlin]

$$\bar{\mathcal{M}}^{(4,0)}_{\text{1-loop}} \neq 0, \quad \bar{\mathcal{M}}^{(3,1)}_{\text{1-loop}} \neq 0, \quad \bar{\mathcal{M}}^{(5,0)}_{\text{1-loop}} \neq 0, \quad \bar{\mathcal{M}}^{(0,5)}_{\text{1-loop}} \neq 0,$$

from soft limits they argue

$$\bar{\mathcal{M}}^{(n,0)}_{\text{1-loop}} = i(n-3)!\delta^{(8)}(\bar{\mathcal{Q}}) \quad \supset \quad \mathcal{M}(h^{++}\, h^{++}\, \bar{t}^{n-2})$$

corresponding to a term in the effective action

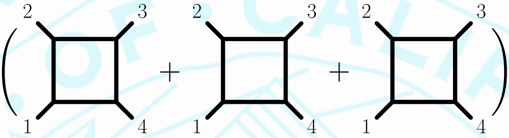
$$i \log(1-\bar{t})(R^+)^2 + \text{ c.c} + \text{ SUSY} = b\, R \wedge R - e^{-\phi} E_4 + \text{ SUSY}$$

but the rest of the anomalous amplitudes are nonlocal.

D-dimensional analysis

- Recalculate with formal polarizations $\rightarrow$ evanescent contributions?

$$i\,s\,t\,\mathcal{A}^{\text{tree}}_{4,\mathcal{N}=4} \times$$



- Organize amplitude in gauge invariant building blocks

$$(F_i F_j F_k F_l) \quad (F_i F_j)(F_k F_l) \quad \text{and} \quad T_{F^3} = -i\,s\,t\,A^{\text{tree}}_{F^3}(1234)$$

with $F_i^{\mu\nu} = k_i^\mu \epsilon_i^\nu - k_i^\nu \epsilon_i^\mu$

- Projection technology [Gehrmann, Glover; Boels]
- Map to gravity

$$F_{i\mu\nu} F_{i\rho\sigma} \rightarrow \frac{2}{\kappa} R_{i\mu\nu\rho\sigma}$$

A surprise

- 4-graviton amplitude

$$\mathcal{M}_4 = \mathcal{M}_{R^2} + \text{ IR} + \text{ other finite}$$

- Evanescent contribution and anomalous pieces have same origin!

$$A_{SYM} \otimes_{\text{KLT}} A_{F^3}$$

[Bern, Edison, Kosower, JPM]

Q: Could it be that both the anomaly and the evanescent pieces are regulator effects that can be removed by local counterterms?

Calculation

We (re)calculate all one-loop anomalous amplitudes for $n = 3, 4, 5$.

$$\bar{M}^{(n,0)} = i(n-3)!\,\delta^{(8)}(\bar{\mathcal{Q}})\,,$$

$$\bar{M}^{(3,1)} = i\frac{\langle 1\,2\rangle[2\,3]\langle 3\,1\rangle}{[1\,2]\langle 2\,3\rangle[3\,1]}\,\delta^{(8)}(\bar{\mathcal{Q}})\,.$$

$$\bar{M}^{(4,1)} = -i\frac{[2\,3][2\,4]s_{34}}{[1\,2]^2[1\,3][1\,4]}\,\delta^{(8)}(\bar{\mathcal{Q}}) + \text{cyclic}(3,4,5)\,,$$

$$\bar{M}^{(3,2)} = i\varepsilon(1,2,3,4)\frac{[3\,4]^2[4\,5]^2[5\,3]^2}{\prod_{i<j}[i\,j]}\,\delta^{(8)}(\bar{\mathcal{Q}})\,,$$

and a few more, using the double-copy.

Note most of them nonlocal.

Local counterterm insertion

- Double-copy for higher dimensional operators [Broedel, Dixon; He, Zhang]

$$A_{YM} \otimes_{\text{KLT}} A_{F^3} \sim M_{\phi^n R^2}$$

- Gives right operator, up to normalization

$$\bar{M}^{(n,0)}_{\text{KLT}} = i(n-2)!\,\delta^{(8)}(\bar{\mathcal{Q}}) \quad \text{vs.} \quad \bar{M}^{(n,0)} = i(n-3)!\,\delta^{(8)}(\bar{\mathcal{Q}})$$

or equivalently

$$\bar{t}^n (R^+)^2 \quad \text{vs.} \quad \frac{\bar{t}^n}{n}(R^+)^2$$

Final result

Insertion of the supersymmetrization of the operator

$$\mathcal{O} = \frac{i}{2}\log(1-\bar{t})(R^+)^2 + \text{ c.c}$$

in all cases gives

$$M_{\mathcal{O}} = -M_{\text{anomalous}}.$$

So all the anomalous amplitudes (local and nonlocal) cancel!

In addition, the evanescent contribution also cancels!

$$\mathcal{O} \supset \frac{1}{2}e^{-\phi}E_4 \sim \frac{1}{2}E_4 + \cdots$$

Conclusion:

All on-shell effects of anomaly and evanescent operators seem to be removable by adding a local counterterm.

Why such operator?

Recall anomaly cancellation in String theory requires

$$H = dB + \omega_{3A} + \omega_{3L}$$

consequently

$$H^2 \supset \omega_{3L} \wedge *dB = \omega_{3L} \wedge db = -b\, R \wedge R + d(\cdots)$$

so $D = 4$ cancellation analogous to Green-Schwarz mechanism.

Addition of this operator very natural if we think of $\mathcal{N} = 4$ SUGRA as the low energy limit of a String theory!

Summary

- Effects of trace anomaly on the divergence not physical
- Duality anomaly is believed to manifest as nonvanishing amplitudes
- In $\mathcal{N} = 4$ SUGRA anomaly and evanescent contributions are closely intertwined
- Effect on $n \leq 5$ amplitudes of both can be removed by adding a local counterterm

Work in progress & future questions

- All *n* argument at one loop
- Higher loop anomalous amplitudes.
- Are there any subtleties off-shell?
- Precise relation to the conformal anomaly?
- What happens to the four loop divergence?

$$\mathcal{M}_4^{4-\text{loop}}\Big|_{\text{div}} = \frac{1}{\epsilon}\frac{(1-264\zeta_3)}{144} st\mathcal{A}_4^{\text{tree}}(\mathcal{O}^{(2,2)} + \mathcal{O}^{(4,0)} + \mathcal{O}^{(3,1)})$$

Thank you!